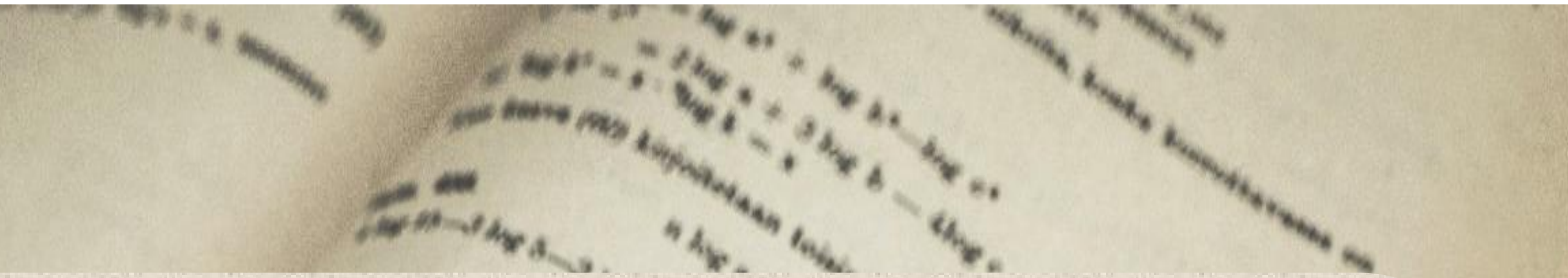
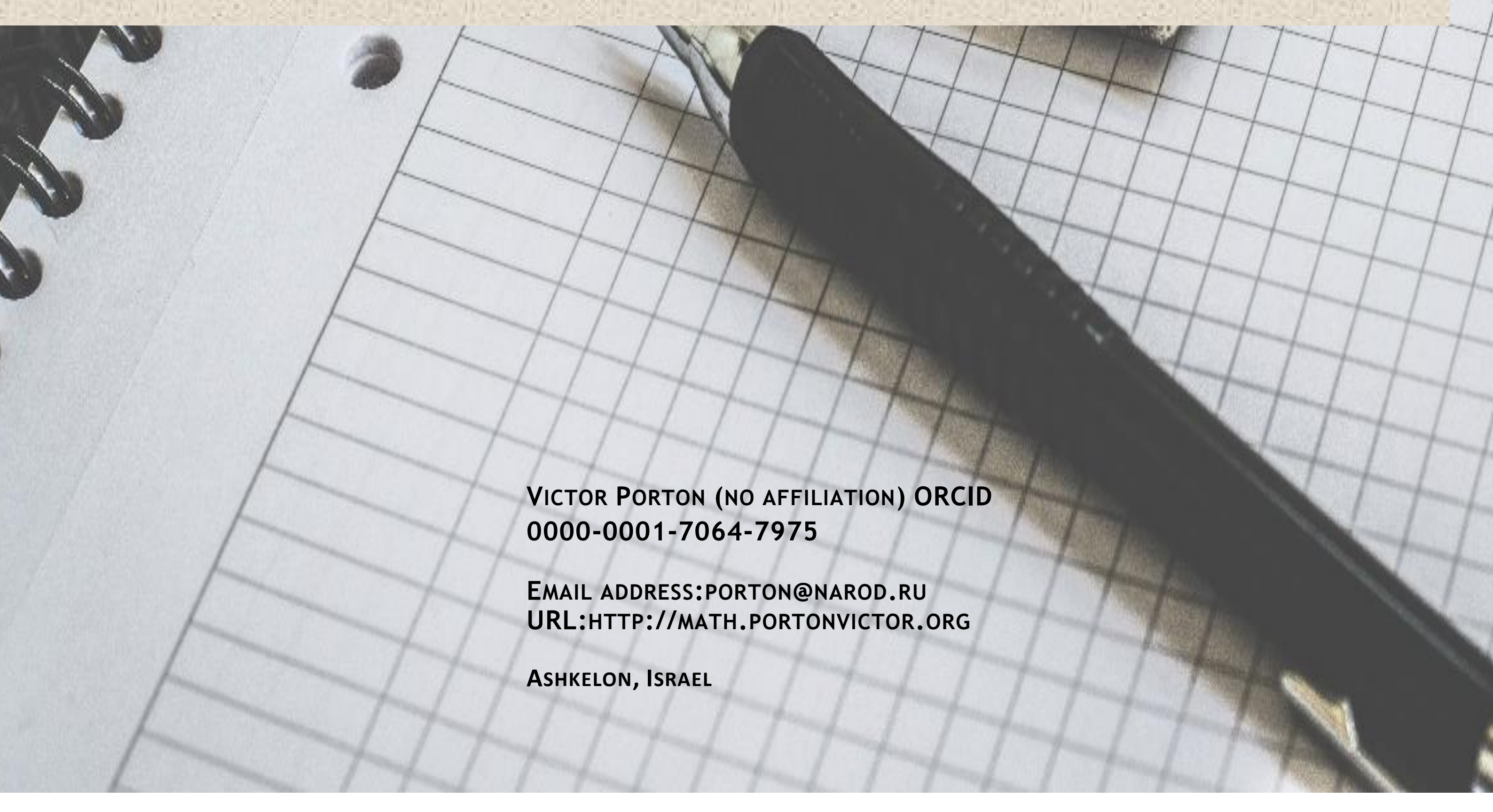


GENERALIZED LIMIT

Arbitrary Discontinuous Function



Victor Porton (no affiliation) ORCID 0000-0001-7064-7975

Email address:porton@narod.ru
URL:http://math.portonvictor.org

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ABSTRACT. I consider (generalized) limit of arbitrary (discontinuous) function, defined in terms of funcoids. Definition of generalized limit makes it obvious to define such things as derivative of an arbitrary function, integral of an arbitrary function, etc. It is given a definition of non-differentiable solution of a (partial) differential equation. It's raised the question how do such solutions "look like" starting a possible big future research program.

The generalized solution of one simple example differential equation is also considered.

The generalized derivatives and integrals are linear operators. For example $\int^b f(x)dx - \int^b f(x)dx = 0$ is defined and true for *every* function.

Contents

1. Introduction

I defined *funcoid* and based on this generalized *limit of an arbitrary (even dis- continuous) function* in [**2**].

In this article I consider generalized limits in more details.

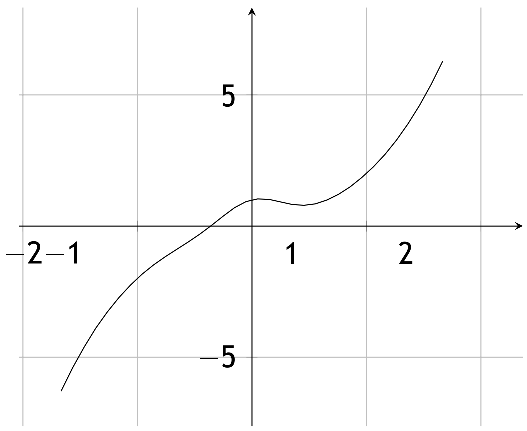
This article is written in such a way that a reader could understand the main ideas on generalized limits without resorting to reading [**2**] beforehand, but to follow the proofs you need read thatfirst.

Definition of generalized limit makes it obvious to define such things as derivative of an arbitrary function, integral of an arbitrary function, etc.

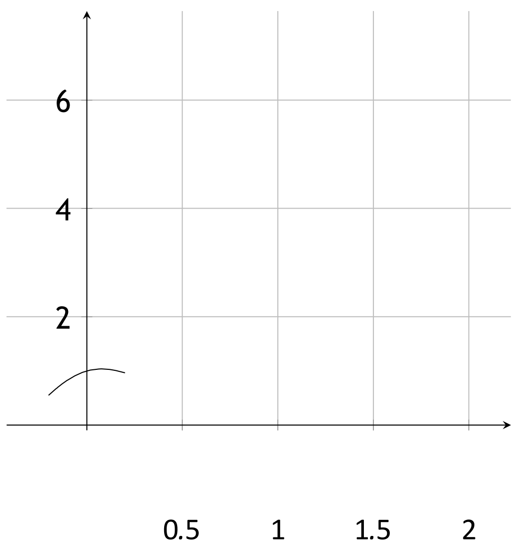
Note that generalized limit is a “composite” object, not just a simple real number, point, or “regular” vector.

2. A popular explanation of generalized limit

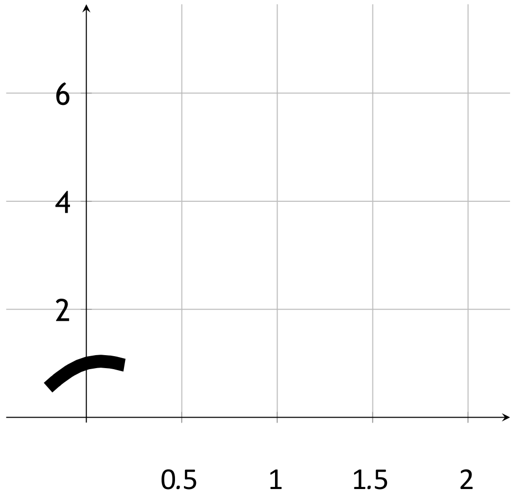
For an example, consider some real function f from x-axis to y-axis:



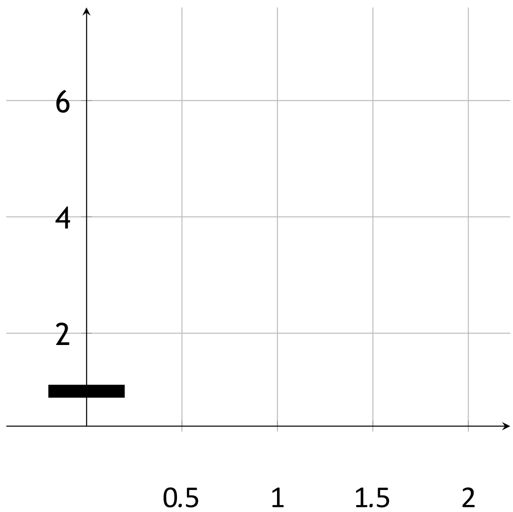
Take it's infinitely small fragment (in our example, an infinitely small interval for x around zero; see below for an explanation what is infinitely small):



Next consider that with a value$y$replaced with an infinitely small interval like $[y-\epsilon;y+\epsilon]$:

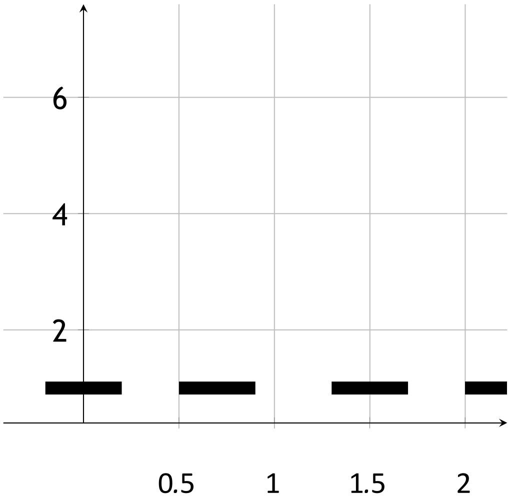


Now we have “an infinitely thin and short strip”. In fact, it is the same as an “infinitely small rectangle” (Why? So infinitely small behave, it can be counter-intuitive, but if we consider the above meditations formally, we could get this result):



This infinitely small rectangle’s $y$position uniquely characterizes the limit of our function (in our example at$x \to 0$).

If we consider the set of all rectangles we obtain by shifting this rectangle by adding an arbitrary number to x, we get



Such sets one-to-one corresponds to the value of the limit of our function (at $x \to 0$): Knowing such the set, we can calculate the limit (take its arbitrary element and get its so to say y-limit point) and knowing the limit value (y), we could write down the definition of this set.

So we have a formula for *generalized limit*:

$$\lim_{x \to a} f(x) = \{ \mathrm{X} \circ f|_{\Delta(a)} \circ r \mid r \in G \}$$

where G is the group of all horizontal shifts of our space $\mathbb{R}$, $f|_{\Delta(a)}$ is the function f of which we are taking limit restricted to the infinitely small interval $\Delta(a)$ around the point a, $\mathrm{X} \circ$ is "stretching" our function graph into the infinitely thin "strip" by applying a topological operation to it.

What all this (especially "infinitely small") means? It is filters and "funcoids" (see below for the definition).

Why we consider all shifts of our infinitely small rectangle? To make the limit not dependent of the point a to which x tends. Otherwise the limit would depend on the point a.

Note that for discontinuous functions elements of our set (our limit is a set) won't be infinitely small "rectangles" (as on the pictures), but would "touch" more than just one y value.

The interesting thing here is that we can apply the above formula to *every* function: for example to a discontinuous function, Dirichlet function, unbounded function, unbounded and discontinuous at every point function, etc. In short, the generalized limit is defined for *every* function. We have a definition of limit for every function, not only a continuous function!

And it works not only for real numbers. It would work for example for any function between two topological vector spaces (a vector space with a topology).

Hurrah! Now we can define derivative and integral of *every* function.